

Calculators, Mobile Phones and Pagers are not allowed

Answer the following questions:

1. Evaluate the following limits, if exist:

(a) $\lim_{x \rightarrow 27} \frac{3x^{\frac{2}{3}} - 27}{x^{\frac{1}{3}} - 3}$

(3 pts.)

(b) $\lim_{\theta \rightarrow 0} \frac{\theta^2}{1 - \cos \theta}$

(3 pts.)

2. State The Intermediate Value Theorem. Prove that the graphs of $f(x) = x^5 + 2x^4 - x^3 + 2x - 1$ and $g(x) = 3x^2 - 2x + 1$, intersect.

(4 pts.)

3. Find the minimum value of $S = 9x + 8y^2$, where $x(y^2 + 1) = 2$ and $x \neq 0$.

(4 pts.)

4. Evaluate the following integrals:

(a) $\int \frac{x^2 + 1}{\sqrt[5]{(x^3 + 3x + 7)^4}} dx$

(4 pts.)

(b) $\int_{-3}^3 (\sqrt{9 - x^2} + x^2 \sin^3 x + x) dx$

(4 pts.)

5. Let $x > 1$, and suppose that the average value of a function f on the interval $[1, x]$, is $f_{av} = x^5$. Find $f(x)$.

(4 pts.)

6. If $f(x) = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$, find the arc length of the graph of f from $x = 0$ to $x = \sqrt{3}$.

(4 pts.)

Find the area of the region bounded by the graphs of the equations $y = x^2$ and $y = x + 2$.

(4 pts.)

7. Let R be the region bounded by the graphs of the equations $y = x^2 + 4$, $2x + y = 2$, $x = 0$ and $x = 1$.

(a) Find the volume of the resulting solid if R is revolved about the line $y = 5$.

(4 pts.)

(b) Set up an integral that can be used to find the volume of the resulting solid if R is revolved about the line $x = 2$.

(2 pts.)